Classification of Multipartite Entanglement via Negativity Fonts

S. Shelly Sharma*
Departamento de Física, Universidade Estadual de Londrina, Londrina 86051-990, PR Brazil

N. K. Sharma†
Departamento de Matemática, Universidade Estadual de Londrina, Londrina 86051-990 PR, Brazil

Partial transposition of state operator is a well known tool to detect quantum correlations between two parts of a composite system. In this letter, the global partial transpose (GPT) is linked to conceptually multipartite underlying structures in a state - the negativity fonts. If $K-$way negativity fonts with non zero determinants exist, then selective partial transposition of a pure state, involving K of the N qubits ($K \leq N$) yields an operator with negative eigevalues, identifying K-body correlations in the state. Expansion of GPT interms of $K-$way partially transposed (KPT) operators reveals the nature of intricate intrinsic correlations in the state. Classification criteria for multipartite entangled states, based on the underlying structure of global partial transpose of canonical state, are proposed. Number of $N-$partite entanglement types for an N qubit system is found to be $2^{N-1} - N + 2$, while the number of major entanglement classes is $2^{N-1} - 1$. Major classes for three and four qubit states are listed. Subclasses are determined by the number and type of negativity fonts in canonical state.

I. INTRODUCTION

Interactions generate correlated systems. Understanding the intricate nature of correlations in multipartite systems is a fundamental problem of physics. Correlations in multipartite systems that need quantum mechanics for their description are conceptually distinct from classical correlations. Entanglement is the best known aspect of such multipartite correlations. Partial transposition of a state operator is a well known tool to detect bi-partite entanglement. In this letter, we link the partial transpose to conceptually multipartite underlying structures - the negativity fonts - and present a new perspective to classification of multipartite entanglement. Selective partial transposition involving a group of subsystems, allows one to detect underlying intricate structure and nature of correlations through the expansion of global partial transpose of the canonical state in terms of K-way partially transposed operators. Each term in the expansion identifies a specific type of multipartite correlations, which may in turn be quantified by constructing appropriate invariantes.

The nature of quantum coherences generated by the interaction determines the entanglement type of a multipartite state. Matrix elements of an N-partite state operator that are off diagonal for K subsystems in a selected basis, determine the $K-$way coherences. Quantum coherences that are present in a GHZ-like state of K two-level (qubit) quantum systems may also be present in an N qubit state, where $2 \leq K \leq N$. We focus on classification of multiqubit entanglement, although, analogous treatment should be valid for subsystems with $d \neq 2$. It was shown in [1] that $K-$way coherences in a multiqubit state can be quantified by partial $K-$way negativities. Partial K-way negativities [2], the contributions of $K-$way coherences to global negativity [3, 4], vary under local operations. How a $K-$way negativity font - the basic unit of $K-$qubit coherences - transforms when one of the qubits undergoes local unitary transformations was pointed out, recently, in [5, 6]. By using local unitaries to express a general $N-$qubit state as a superposition of minimum number of local basis product (LBP) states, one arrives at a canonical state with characteristic number and type of negativity fonts quantifying inherent $K-$way coherences ($2 \leq K \leq N$). The key idea is to explore the inevitable link between the quantum coherences and entanglement manifest in the global partial transpose (GPT) [7]. All states with the same type of intrinsic quantum coherences belong to the same class. Furthermore, the number of negativity fonts with non-zero determinants, in a $K-$way partially transposed canonical state, is fixed. Therefore, the states in a given class can be grouped together into subclasses depending on the number and type of distinct negativity fonts with non zero determinants in global partially transposed canonical state.

Two multipartite pure states are considered equivalent under stochastic local operations and classical communication (SLOCC), if one can be obtained from the other with some probability using only local operations and classical communication among different parties. Classification of three qubit states focussed on local unitary invariance in

*Electronic address: shelly@uel.br
†Electronic address: nsharma@uel.br

ref. [8], and SLOCC equivalence in [9]. Classification of four qubit states into nine families in [10] and more recently from string theory approach in [11] has been expanded by Li et al. [12] to 49 SLOCC entanglement classes. Lamata et al. [13] use an inductive approach [14] to classify four-qubit pure states. Their main result is that each of the eight genuine inequivalent entanglement classes contains a continuous range of strictly nonequivalent states, although with similar structure. It is clear that as we go to multipartite systems (or higher dimensional systems), we are more likely to encounter continuous range of strictly nonequivalent states, although with similar structure. In this context, our classification criteria based on the nature of quantum correlations, offer a distinct logical perspective. Local unitary invariants, obtained from transformation equations for determinants of negativity fonts under local unitaries [6], or SLOCC [15] quantify the entanglement of states in a sub class. Classification of three and four qubit states is reviewed.

II. BASIC CONCEPTS

In order to clearly state our classification criteria, we introduce some notations and definitions. A general $N-$qubit pure state reads as

$$\left|\Psi^{A_1,A_2,...A_N}\right\rangle = \sum_{i_1 i_2 ... i_N} a_{i_1 i_2 ... i_N} \left| i_1 i_2 ... i_N \right\rangle \tag{1}$$

where $\left| i_1 i_2 ... i_N \right\rangle$ is a local basis product (LBP) state in 2^N dimensional Hilbert space, and A_p is the location of qubit p. The coefficients $a_{i_1 i_2 ... i_N}$ are complex numbers. The local basis states of a single qubit are labelled by $i_m = 0$ and 1, where $m = 1, ..., N$. The global partial transpose $\widehat{\rho}_G^{T_{A_p}}$ [7] with respect to qubit p is constructed from N qubit state operator $\widehat{\rho}=\left|\Psi\right\rangle \left\langle\Psi\right|$ by partial transposition

$$\left\langle i_1 i_2 ... i_N \right| \widehat{\rho}_G^{T_{A_p}} \left| j_1 j_2 ... j_N \right\rangle = \left\langle i_1 i_2 ... i_{p-1} j_p i_{p+1} ... i_N \right| \widehat{\rho} \left| j_1 j_2 ... j_{p-1} i_p j_{p+1} ... j_N \right\rangle . \tag{2}$$

The $K-$way partial transpose, $\widehat{\rho}_K^{T_{A_p}}$, obtained by selective transposition is defined as

$$\left\langle i_1 i_2 ... i_N \right| \widehat{\rho}_K^{T_{A_p}} \left| j_1 j_2 ... j_N \right\rangle = \left\langle i_1 i_2 ... i_{p-1} j_p i_{p+1} ... i_N \right| \widehat{\rho} \left| j_1 j_2 ... j_{p-1} i_p j_{p+1} ... j_N \right\rangle , \tag{3}$$

if $\sum\limits_{m=1}^{N} (1 - \delta_{i_m,j_m}) = K, \quad \text{and} \quad \delta_{i_p,j_p} = 0$, while

$$\left\langle i_1 i_2 ... i_N \right| \widehat{\rho}_K^{T_p} \left| j_1 j_2 ... j_N \right\rangle = \left\langle i_1 i_2 ... i_N \right| \widehat{\rho} \left| j_1 j_2 ... j_N \right\rangle ,$$

if $\sum\limits_{m=1}^{N} (1 - \delta_{i_m,j_m}) \neq K$. Decomposition of global partial transpose, $\widehat{\rho}_G^{T_{A_p}}$, of an $N-$qubit state (Eq. (1)) with respect to qubit A_p in terms of $K-$way partially transposed operators reads as

$$\widehat{\rho}_G^{T_{A_p}} = \sum_{K=2}^{N} \widehat{\rho}_K^{T_{A_p}} - (N-2)\widehat{\rho}. \tag{4}$$

Expansion of GPT interms of $K-$way partially transposed (KPT) operators has information about the nature of intricate intrinsic correlations in the state.

Negativity fonts - Following the notation used in ref. [6], an $N-$way negativity font and it's determinant read as

$$\nu^{i_1 i_2 ... i_p ... i_N} = \begin{bmatrix} a_{i_1 i_2 ... i_p ... i_N} & a_{i_1+1, i_2+1, ... i_p ... i_N+1} \\ a_{i_1 i_2 ... i_p+1 ... i_N} & a_{i_1+1, i_2+1, ... i_p+1 ... i_N+1} \end{bmatrix}, D^{i_1 i_2 ... i_p ... i_N} = \det\left(\nu^{i_1 i_2 ... i_p ... i_N}\right),$$

when the state of qubit p is involved in partial transposition. Here $i_m + 1 = 0$ for $i_m = 1$ and $i_m + 1 = 1$ for $i_m = 0$. When global partial transpose is expressed as a sum of $K-$way partialy transposed operators, the negativity fonts are distributed over $N-1$ operators. Since K qubits may be chosen in $\left(\frac{N!}{(N-K)!K!}\right)$ ways, the form of a $K-$way font must specify the set of K qubits it refers to. To distinguish between different $K-$way negativity fonts a list of qubit states for which $\delta_{i_m,j_m} = 1$ appears as a subscript of ν. In other words, a $K-$way font involving qubits A_{q+1} to A_{q+K} such that $\sum\limits_{m=1}^{N} (1 - \delta_{i_m,j_m}) = \sum\limits_{m=q+1}^{q+K} (1 - \delta_{i_m,j_m}) = K$ reads as

$$\begin{aligned} &\nu^{i_1 i_2 ... i_p ... i_N}_{(A_1)_{i_1}, (A_2)_{i_2}, ... (A_q)_{i_q} (A_{q+K+1})_{i_{q+K+1}} ... (A_N)_{i_N}} \\ &= \begin{bmatrix} a_{i_1 i_2 ... i_p ... i_N} & a_{i_1 i_2 ... i_q, i_{q+1}+1, i_{q+2}+1, ... i_p ..., i_{q+K}+1, i_{q+K+1}, ... i_N} \\ a_{i_1 i_2 ... i_p+1 ... i_N} & a_{i_1 i_2 ... i_q, i_{q+1}+1, i_{q+2}+1, ... i_p+1 ..., i_{q+K}+1, i_{q+K+1}, ... i_N} \end{bmatrix}, \end{aligned}$$

and its determinant is represented by $D^{i_{q+1}i_{q+2}...i_p...i_{q+k}}_{(A_1)_{i_1},(A_2)_{i_2},...(A_q)_{i_q}(A_{q+K+1})_{i_{q+K+1}}...(A_N)_{i_N}}$. If $K-$way negativity fonts with non zero determinants exist, then selective partial transposition of a pure state, involving K of the N qubits $(K \leq N)$ yields an operator with negative eigevalues, identifying $K-$body correlations in the state. Local unitaries on qubit A_r with $r \neq p$, yield transformation equations relating determinants of $K-$way and $(K \pm 1)-$way negativity fonts. Consequently, the contribution of partial $K-$way negativities to global negativity varies with local unitaries.

III. CANONICAL STATE

A unitary transformation $\mathrm{U}^{A_1} = \frac{|a_{11J}|}{\sqrt{|a_{01J}|^2+|a_{11J}|^2}} \begin{bmatrix} 1 & -\frac{a_{01J}}{a_{11J}} \\ \frac{a^*_{01J}}{a^*_{11J}} & 1 \end{bmatrix}$, followed by an invertible local operator transforms an N-way negativity font in $\rho_N^{T_{A_1}}$ as

$$\begin{bmatrix} a_{00I} & a_{01J} \\ a_{10I} & a_{11J} \end{bmatrix} \overset{U^{A_1}}{\longrightarrow} \begin{bmatrix} b_{00I} & 0 \\ b_{10I} & b_{11J} \end{bmatrix} \overset{O^{A_1}}{\longrightarrow} \begin{bmatrix} 1 & 0 \\ 0 & c_{11J} \end{bmatrix}$$

such that $b_{00I}b_{11J} = a_{00I}a_{11J} - a_{10I}a_{01J}$ and entanglement type of the state is not changed. Here, I stands for the string $\{i_3 i_4 ... i_N\}$. Therefore, the most general $N-$qubit state with 2^{N-2} $N-$way fonts can be transformed by LOCC to a unitary equivalent canonical form which is a superposition of $2^N - N$ LBP States. The state having information about the entanglement type has at the most $2^N - 2N$ LBP States.

A unitary transformation U^{A_q} (applied to a qubit other than A_1) may be selected such that

$$\det \begin{bmatrix} a_{00I} & a_{01J} \\ a_{10I} & a_{11J} \end{bmatrix} \overset{U^{A_q}}{\longrightarrow} \det \begin{bmatrix} c_{00I} & c_{01J} \\ c_{10I} & c_{11J} \end{bmatrix} = 0$$

thus annihilating a negativity font. In an entangled state, the maximum number of negativity fonts that may be annihilated by unitary operations and classical communication is $N-1$. The final canonical state obtained after local invertible operations and classical communication will have a variable number of negativity fonts and LBP states. For states with $N > 3$ more than one representations of canonical state may be possible.

IV. CRITERION FOR CLASSIFYING N-QUBIT ENTANGLED STATES

A logical criterion for classification of multipartite entanglement can be based on the structure of global partial transpose (Eq. (4)) of canonical state and number of $K-$way negativity fonts. Determinants of negativity fonts are the intrinsic negative eigenvalues of partially transposed state operator. No wonder, squared global negativity is four times the sum of squared moduli of determinants of all negativity fonts [16] available for a given qubit. Combinations of Determinants of Negativity fonts determine the local unitary invariants characterizing an N-qubit state [5, 6]. These observations lead us to a classification scheme in which

(a) An entanglement class is characterized by the set of $K-$way $(2 \leq K \leq N)$ partially transposed operators present in the expansion of global partial transpose of the canonical state $\widehat{\rho}_c$. Global partial transpose of canonical state has the decomposition $(\widehat{\rho}_c)_G^{T_{A_p}} = \sum\limits_{K=2}^{N} (\widehat{\rho}_c)_K^{T_{A_p}} - (N-2)\widehat{\rho}_c$. Since K varies from 2 to N, the number of possible combinations of $K-$way partially transposed matrices is

$$N_{Class} = \sum_{m=1}^{N-1} \frac{(N-1)!}{m!\,(N-m-1)!} = \left(2^{N-1} - 1\right),$$

which is also the number of major entanglement classes. An $N-$partite state is said to be $N-$partite entangled if and only if all bipartite partitions produce mixed reduced density matrices. Besides the fully separable and fully entangled, there also exists classes of states that are partially separable. The case $\widehat{\rho}_G^{T_{A_p}} = \widehat{\rho}_K^{T_{A_p}}$ results in an $N-$partite entangled state, only if $K = N$ or $K = 2$. Therefore the number of $N-$partite entanglement types is $\left(2^{N-1} - N + 2\right)$. States obtained by permuting the qubits are grouped in the same class.

(b) States with similar decomposition of $(\widehat{\rho}_c)_G^{T_{A_p}}$ with respect to different number of qubits belong to different sub classes or families in the same class. For a given qubit, the number of $K-$way negativity fonts in a $K-$way partially transposed matrix varies from 0 to 2^{N-2}. The number and types of non-zero determinants of distinct negativity fonts in KPT canonical state operators characterize nonequivalent sub classes of states.

TABLE I: Classification of three qubit States

Class	Decomposition of $(\rho_c)_G^{T_{A_p}}$	Representative ρ_c	$\{N_{3-way}, N_{2-way}\}$	3-tangle
CI	$(\rho_c)_3^{T_{A_p}} + (\rho_c)_2^{T_{A_p}} - \widehat{\rho}_c$	$\|000\rangle + \|111\rangle + \|110\rangle$	$\{1,1\}$,$\{1,2\}$,$\{1,3\}$	$\tau_3 \neq \left(N_G^{A_p}\right)^2$
CII	$(\rho_c)_3^{T_{A_p}}$	$\|000\rangle + \|111\rangle$	$\{1,0\}$	$\tau_3 = \left(N_G^{A_p}\right)^2$
CIII	$(\rho_c)_2^{T_{A_p}}$	$\|000\rangle + \|110\rangle + \|101\rangle$	$\{0,1\}$,$\{0,2\}$	$\tau_3 = 0$

(c) The values of entanglement monotones based on relevant local unitary polynomial invariants quantify the entanglement of a state in a sub-class.

Special Entanglement types of N–qubit States - From criterion (a), it is natural to find states with similar entanglement type but different number of qubits. For instance, all N- qubit GHZ like states are characterized by $(\widehat{\rho}_c)_G^{T_{A_p}} = (\widehat{\rho}_c)_N^{T_{A_p}}$, while for N qubit W-like states $(\widehat{\rho}_c)_G^{T_{A_p}} = (\widehat{\rho}_c)_2^{T_{A_p}}$. Other entanglement types can be identified by using the transformation properties of determinants of negativity fonts and unitary equivalence. For example, a common feature of a special set of states with $(\widehat{\rho}_c)_G^{T_{A_p}} = (\widehat{\rho}_c)_N^{T_{A_p}} + (\widehat{\rho}_c)_2^{T_{A_p}} - \widehat{\rho}_c$ is a unitary equivalent description with $(\widehat{\rho}_c)_G^{T_{A_p}} = (\widehat{\rho}_c)_{N-1}^{T_{A_p}} + (\widehat{\rho}_c)_2^{T_{A_p}} - \widehat{\rho}_c$. To illustrate, we consider, the global partial transpose of an $N-$qubit state with canonical form

$$|\Psi\rangle = a_{00...0}\,|00...0\rangle + a_{11...1}\,|11...1\rangle + a_{0011...1}\,|0011...1\rangle + a_{1100...0}\,|1100...0\rangle\,, \tag{5}$$

which is characterized by $N-$way, $(N-2)$-way and $2-$way negativity fonts. The combination $\left(D^{00...0} - D^{0011...1}\right)^2 - 4D^{00...0}_{(A_2)_0}D^{00...0}_{(A_2)_1}$ is invariant with respect to local unitaries on qubits A_1 and A_2. If the determinants of two $N-$way fonts satisfy, $D^{00...0} + D^{0011...1} = 0$,then the form of invariant indicates the existence of local unitaries on A_1 and A_2 that transform the state to a form for which GPT has $(N-1)$ $-$way negativity fonts. For the state of Eq. (5) the unitary equivalent form reads as

$$|\Psi'\rangle = b_{00...0}\,|00...0\rangle + b_{1011...1}\,|1011...1\rangle + b_{0111...1}\,|0111...1\rangle + b_{1100...0}\,|1100...0\rangle\,.$$

Four qubit cluster state $|C\rangle = |0000\rangle - |0011\rangle + |1110\rangle + |1101\rangle$,belongs to the class with $(\widehat{\rho}_c)_G^{T_{A_p}} = (\widehat{\rho}_c)_4^{T_{A_p}} + (\widehat{\rho}_c)_2^{T_{A_p}} - \widehat{\rho}$. In general, when two equivalent canonical descriptions are possible the one with higher order negativity fonts should determine the class.

V. COUNTING THE CLASSES AND SUB CLASSES

For the most general three qubit state, local unitaries yield the canonical form[8]

$$|\Psi_c\rangle = a_{000}\,|000\rangle + a_{100}\,|100\rangle + a_{111}\,|111\rangle + a_{101}\,|101\rangle + a_{110}\,|110\rangle\,, \tag{6}$$

having a single $3-$way negativity font in $\widehat{\rho}_G^{T_{A_1}}$ and five LBPS. Global partial transpose of $\widehat{\rho} = |\Psi_c\rangle\langle\Psi_c|$ has the decomposition $(\widehat{\rho}_c)_G^{T_p} = (\widehat{\rho}_c)_3^{T_p} + (\widehat{\rho}_c)_2^{T_p} - (\widehat{\rho}_c)$. Table I displays the three major classes, distinguished by the relation between

$$\tau_3 = 4\left|\left(\left(D^{000} + D^{001}\right)^2 - 4D^{00}_{(A_3)_0}D^{00}_{(A_3)_1}\right)\right|,$$

and global negativity of GPT, along with $\{N_{3-way}, N_{2-way}\}$ that characterize the sub-classes of three qubit entangled states. Here N_{K-way} is the number of $K-$way negativity fonts with non-zero determinants.

Applying the classification criterion (a) to four qubit states we obtain seven major classes of states of which six contain states with four qubits entangled to each other. Table II. displays a representative of each class and possible number of $\{N_{4-way}, N_{3-way}\}$. Sub classes in a major class have different number of LBP states and characteristic combination of $\{N_{4-way}, N_{3-way}, N_{2-way}\}$ in canonical state. To distinguish between the states in the same sub-class, one resorts to four qubit invariants. We notice that states with four-partite entanglement arise in six of the seven classes that is there are six distinct ways of entangling four qubits. In comparison, Lamata claim eight distinct entanglement types and Acin et al. [8] claim nine distinct entanglement types. The observation of Lamata et al. [13] that the class $L_{0_{3\oplus1}0_{3\oplus1}}$ [8] with canonical state $|0000\rangle + |1110\rangle$ does not contain genuinely entangled four qubit states is

TABLE II: Seven Classes of Four qubit States.

Class	Decomposition of $(\rho_c)_G^{T_{A_p}}$	Representative $\widehat{\rho}_c$	$\{N_{4-way}, N_{3-way}\}$
CI	$(\rho_c)_4^{T_{A_p}} + (\rho_c)_3^{T_{A_p}} + (\rho_c)_2^{T_{A_p}} - 2\widehat{\rho}_c$	$\|0000\rangle + \|1111\rangle + \|1110\rangle + \|1100\rangle$	$\{(3-1), (1-12)\}$
CII	$(\rho_c)_4^{T_{A_p}} + (\rho_c)_3^{T_{A_p}} - \widehat{\rho}_c$	$\|0000\rangle + \|1111\rangle + \|1110\rangle$	$\{1, 1\}$
CIII	$(\rho_c)_4^{T_{A_p}} + (\rho_c)_2^{T_{A_p}} - \widehat{\rho}_c$	$\|0000\rangle + \|1111\rangle + \|0011\rangle - \|0011\rangle$	$\{(4-2), 0\}$
CIV	$(\rho_c)_3^{T_{A_p}} + (\rho_c)_2^{T_{A_p}} - \widehat{\rho}_c$	$\|0000\rangle + \|1110\rangle + \|1101\rangle$	$\{0, 1-4\}$
CV	$(\rho_c)_4^{T_{A_p}}$	$\|0000\rangle + \|1111\rangle$	$\{1, 0\}$
CVI	$(\rho_c)_3^{T_{A_p}}$	$\|0000\rangle + \|1110\rangle$; (only Separable)	$\{0, 1\}$
CVII	$(\rho_c)_2^{T_{A_p}}$	$\|0000\rangle + \|1100\rangle + \|1010\rangle + \|1001\rangle$	$\{0, 0\}$

consistent with class VI in table II. In addition, we notice that the states with canonical forms $|0000\rangle + |1111\rangle + |1100\rangle$ and $|0000\rangle + |1111\rangle + \lambda_1 |1100\rangle + \lambda_2 |0011\rangle$ differ only in the number of four way negativity fonts, while both have a canonical form with $(\widehat{\rho}_c)_G^{T_p} = (\widehat{\rho}_c)_4^{T_p} + (\widehat{\rho}_c)_2^{T_p} - (\widehat{\rho}_c)$. Like wise the states with $(\widehat{\rho}_c)_G^{T_p} = (\widehat{\rho}_c)_4^{T_p} + (\widehat{\rho}_c)_3^{T_p} + (\widehat{\rho}_c)_2^{T_p} - 2(\widehat{\rho}_c)$, but different number of negativity fonts belong to different sub-classes in our classification, whereas are classified as different entanglement types by Lamata et al. [13] (span $\{0_k\Psi, GHZ\}$, some of the states from span $\{0_k\Psi, 0_k\Psi\}$ and span $\{GHZ, W\}$). For five qubits States one obtains fifteen classes of which 13 contain states with all five qubits entangled to each other.

VI. CONCLUSIONS

Using the structure of global partial transpose of the canonical state as a key to the nature of intrinsic quantum correlations, we have proposed criteria for classifying N qubit pure states. The set of criteria offers a new perspective to entanglement classification and is extendible to mixed state entanglement. It has been pointed out that for different number of qubits one may identify states with similar quantum correlations. Most of the major classes contain a continuous range of strictly SLOCC nonequivalent states with similar structure. The number of genuine entanglement types for a given value of N is easily counted. As the number of $N-$way negativity fonts increases, so does the number of sub-classes. However, the distinct number of new entanglement types does not grow so fast. It will be interesting to study the new features, such as suitability for a given QIP task, that appear with increasing N for states with same type of intrinsic quantum correlations.

This work is supported by CNPq Brazil and FAEP UEL Brazil.

[1] S. S. Sharma and N. K. Sharma, Phys. Rev. A 76, 012326 (2007).
[2] S. S. Sharma and N. K. Sharma, Phys. Rev. A 78, 012113 (2008); S. S. Sharma and N. K. Sharma, Phys. Rev. A 79, 062323 (2009).
[3] K. Zyczkowski, P. Horodecki, A. Sanpera, and M. Lewenstein, Phys. Rev. A 58, 883 (1998).
[4] G. Vidal and R. F. Werner, Phys. Rev. A 65, 032314 (2002).
[5] S. S. Sharma and N. K. Sharma, Phys. Rev. A 82, 012340 (2010).
[6] S. S. Sharma and N. K. Sharma, Phys. Rev. A 82, 052340 (2010).
[7] A. Peres, Phys. Rev. Lett. 77, 1413 (1996).
[8] A. Acin, A. Andrianov, L. Costa, E. Jane, J. I. Latorre, and R. Tarrach, Phys. Rev. Lett. 85, 1560 (2000).
[9] W. Dür, G. Vidal, and J. I. Cirac, Phys. Rev. A 62, 062314 (2000).
[10] F. Verstraete, J. Dehaene, B. DeMoor, and H. Verschelde, Phys. Rev. A 65, 052112 (2002).
[11] L. Borsten, D. Dahanayake, M. J. Duff, A. Marrani and W. Rubens, Phys. Rev. Lett.105, 100507 (2010).
[12] D. Li, X. Li, H. Huang, and X. Li, Quant. Inf. Comp. 9, 0778 (2009).
[13] L. Lamata, J. León, D. Salgado and E. Solano, Phys. Rev. A 75, 022318 (2007).
[14] L. Lamata, J. León, D. Salgado and E. Solano, Phys. Rev. A 74, 052336 (2006).
[15] D. Li, X. Li, H. Huang, X. Li, Phys. Rev. A 76, 032304 (2007).
[16] S. S. Sharma and N. K. Sharma, 'Negativity fonts, multiqubit invariants and four qubit maximally entangled states', Quantum Information Processing, published online (11 Nov. 2011), doi 10.1007/s11128-011-0324-0.